A GUIDELINE for the VALIDATION of COMPUTATIONAL SOLID MECHANICS MODELS using FULL-FIELD OPTICAL DATA

A methodology for the comparison of data-rich maps from simulations with those from experiments for the purpose of validating a computational solid mechanics model. Prepared as part of the ADVISE project[*] (Grant Agreement SCP7-GA-2008-218595); a shared cost RTD project with the European Commission's Safety and Security by Design Programme (FP7-SST-2007-RTD-1)

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Executive Summary

This guideline has been prepared by the ADVISE (Advanced Dynamic Validations using Integrated Simulation and Experimentation) consortium of university research laboratories, national laboratories, instrument/system manufacturers and transportation vehicle manufacturers. It is intended to supplement the existing Guide for Verification and Validation in Computational Solid Mechanics, published by the American Society of Mechanical Engineers (ASME) in 2006, by describing a process for validating models of structural components using full-field strain data from optical methods of strain measurement. The use of image decomposition to describe strain fields based on feature vectors is recommended in order to achieve reduced dimensionality. This approach enables a simple comparison of data-rich strain fields from a computational model and a validation experiment to be made utilising the uncertainty to assess the acceptability of the correlation. The procedure is described in the context of static loading of pseudo-planar surfaces of engineering artefacts. Its extension to 'snap-shots' from dynamic loading is straightforward if the results are synchronized in time. It is anticipated that the procedure described here would be part of a verification and validation plan as outlined in the existing ASME guide.

1. Introduction

1.1. Overview

The verification and validation of simulations conducted in computational solid mechanics has been identified as an essential step in the process of design analysis[1]. In this context validation has been defined as "the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model". However, there is little guidance offered on the form or methodology for such a process. This guideline is intended to fill this gap for the analysis of structural components. The process focuses on the quantitative comparison of data-rich optical measurements with the results from a computational model. The acceptability of the comparison is discussed in the context of the uncertainties associated with the data from the model and the experiment, and with the comparison process. However, a prescription for validating a specific model is not provided since it is anticipated that the procedure described here would be incorporated into a verification and validation plan of the type described in the ASME Guide for Verification and Validation in Computational Solid Mechanics[1].

1.2. Background

The guideline described here is the result of an international collaboration called the ADVISE (Advanced Dynamic Validation through Integrated Simulations and Experimentation) project involving university research laboratories, national laboratories, instrument manufacturers and end-users from six countries. In addition, during the preparation of this guideline input was sought from the international engineering community through VAMAS TWA26 [2] and the Society for Experimental Mechanics. It is intended that this guideline should supplement those published previously by the ASME[1]. It is the intention of the ADVISE consortium to submit these guidelines for approval by VAMAS and ISO so that they can become established as a precursor to an international standard.

The focus of ADVISE has been on dynamic loading cases which represents a progression from its precursor the SPOTS (Standardisation Project for Optical Techniques of Strain measurement) project that focused on pseudo-static in-plane loading. A set of guidelines on the Calibration and Evaluation of Optical Systems for Strain Measurement[3] were produced based on the SPOTS project. Calibration involves making comparisons with a known, recognised reference material which in turn has been compared via a continuous chain of comparisons to an international standard. The standard metre appears to be the

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[1] ASME V&V 10-2006, Guide for verification and validation in computational solid mechanics, American Society of Mechanical Engineers, New York, 2006.

[2] http://www.vamas.org/twa26

[3] Guidelines for the Calibration & Evaluation of Optical Systems for Strain Measurement, www.opticalstrain.org, ISBN 978-0-9842142-2-8, 2010

most appropriate primary standard for strain. The concepts of standardisation and traceability are discussed in more detail by Hack et al.[4]

For the validation of pseudo-static cases the SPOTS calibration procedure can be combined with the methodology described in these guidelines for the comparison of full-field data from simulations and experiments. The validation process described below has been conceived for data acquired from planar, or pseudo-planar regions of engineering artefacts. Its application to dynamic cases would be straightforward when 'snap-shots' of data are acquired during a dynamic event using a suitably calibrated measurement system. The extension of the process to strain fields acquired from strongly-curved surfaces is not straightforward.

2. Calibration requirements

There will be uncertainties associated with all data acquired from experiments which will arise from a number of sources. An appropriate calibration procedure will permit the evaluation of the minimum measurement uncertainty that can be achieved with a particular experimental set-up, which is equivalent to the uncertainty in the calibration. A set of guidelines are available for the calibration of optical systems for strain measurement[2] within a framework that allows traceability to be established to the international standard for length. At the moment, the reference material, recommended in the guidelines for calibration, is suitable for pseudo-static loading cases on plane, or pseudo-plane surfaces.

It is recommended that the outputs reported from a calibration should be:

(a) the calibration factor or calibration curve relating the instrument output to the strain value of the reference material. Note that the calibration should be performed for the component of strain that it is intended to use in the validation process.

(b) the field of deviations between the predicted and measured values of strain, ε, in the Reference Material, $d(i,j)$ over the gauge area for each point (i,j)

$$d(i,j) = \left(\varepsilon(x_i, y_j)\right)_{predicted} - \left(\varepsilon(x_i, y_j)\right)_{measured} \tag{1}$$

for the appropriate load.

(c) the calibration uncertainty, $u_{cal}(\varepsilon)$ which can be evaluated as

$$u_{cal}(\varepsilon) = \sqrt{u^2(d) + u^2_{RM}(\varepsilon)} \tag{2}$$

where $u_{RM}(\varepsilon)$ is the reference material uncertainty and will have a number of components associated with its material properties, and the accuracy of

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[4] Hack, E., Burguete, R.L., Patterson, E.A., 'Traceability of optical techniques for strain measurement', *Appl. Mechanics and Materials*, 3-4, 391-396, 2005.

manufacture amongst other factors. $u(d)$ is the uncertainty associated with the measurements made of the strain in the Reference Material and is calculated from the field of deviations, Eq. (1).

3. Validation

3.1. Overview

In many areas, it has been common practice to validate computational solid mechanics models using data from a single strain gauge or a set of strain gauges located in the region of maximum stress predicted by the model. This is simple and low in cost, but leaves results from the model not validated for the majority of the spatial domain with the possibility that, despite agreement at the location of the strain gauge, another larger stress is present elsewhere in the prototype and not predicted by the model. It also exposes a risk associated with removing material from the design in areas of predicted low or zero stress in order to save weight. Consequently, it is recommended that *validation of computational solid mechanics models, intended for use in predicting structural integrity, should be performed using full-field maps of surface strain [R1].*

The advent of full-field methods of strain evaluation, utilizing non-contact optical techniques based on digital technology, provides the opportunity for a more comprehensive approach to be taken to the validation of computational solid mechanics models. Now, it is relatively easy and cheap to obtain deformation fields defined over the entire surface of engineering artefacts, by using techniques such as digital image correlation (DIC), digital speckle pattern interferometry (DSPI) and thermoelastic stress analysis (TSA). These techniques can be used to generate data-rich fields of strain that might contain of the order of 10^6 data points, which is comparable to the number of individual elements in a finite element model. Thus, in experiments, it is feasible to acquire strain data over the entire surface of an artefact; and, such a data field should provide the very strong evidence for validating a computational solid mechanics model. The data should be acquired using a calibrated instrument with a sufficiently low calibration uncertainty. However, in practice the surface may need to be sub-divided: to avoid obstructions to optical access; to achieve pseudo-planar conditions in the field of view; and to ensure sufficient spatial resolution. The latter two factors are important in reducing measurement uncertainties. While, the proportion of the surface area of the artefact over which strain data should be validated will depend on the purpose for which it is intended to employ the model; it is recommended that *strain data should be acquired from the entire surface to which optical access can be achieved and that the surface be sub-divided as necessary to reduce measurement uncertainties [R2]*.

The restriction to surface strain is appropriate because of the lack of readily-available techniques for measuring strain in the interior of an engineering artefact. Noting that techniques such as three-dimensional photoelasticity are only applicable to transparent materials and otherwise require models, and x-ray computed tomography is limited by its cost and the size of the object that can be examined. Strain is specified, rather than displacement, because displacement could contain rigid-body components that are not related directly to the structural performance of the engineering artefact, and it is assumed that an assessment of the structural performance of a prototype design is the primary purpose for using a computational solid mechanics model. In this context, it is recommended that *maximum principal strain is the most relevant component of strain to be employed for validation purposes [R3].* However, when a convoluted strain quantity, such as the first invariant, is the output from the measurement system then it is more appropriate to utilise the convoluted quantity than to introduce additional uncertainty by de-convoluting it.

It is good practice to conduct experiments specially designed for the purpose of generating data for a validation process. An experiment can be considered as a physical model of reality since usually it contains some level of idealisation in order to render it practical to conduct. The level of idealisation should be reduced to a minimum through the use of prototypes that come as close as possible to the manufactured artefact in terms of geometry, material and scale; and should be used with loading and boundary conditions that reproduce those strain levels anticipated in service. There are many texts covering the topic of the design of experiments that can be consulted[5].

The remainder of this chapter is divided into two parts that describe the steps in the validation process. First, a method is described for compressing the datasets obtained from the experiment and model. Second, a method for correlating the compressed data in a quantitative manner is introduced and the acceptability, or otherwise of the correlation is discussed. The complete process is shown schematically in figure 1. The need to validate models of sub-components, components, sub-elements, elements and the complete system, in a bottom-up approach, is described in ASME V&V 10-2006[1], and so is not discussed here; however, the process in figure 1 can be applied repeatedly to a hierarchical set of models.

3.2. Image Decomposition

In general, strain fields obtained from experiments and computational models will be data-rich, i.e. containing strain data at more than 10^4 points, will be defined in different co-ordinate systems and in arrays with different pitches, and will be orientated differently, for instance, as a result of the location of the sensor in the experiment. These factors render the direct comparison of two strain fields impractical on a point-by-point basis. A practical alternative is to consider the strain fields as images in which the level of the strain is represented by the grey level values of the image. Then, these images can be decomposed to

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[5] e.g. Anthony, J., 2003, *Design of Experiments for Engineers and Scientists*, Butterworth-Heinemann, Oxford.

feature vectors containing typically less than 10^2 shape descriptors; and, a quantitative comparison made of the feature vectors. Typically, shape descriptors are the coefficients of orthogonal polynomials used to describe the image; and thus, for a specified set of appropriate polynomials, contain the information required to describe uniquely the essential features of the image.

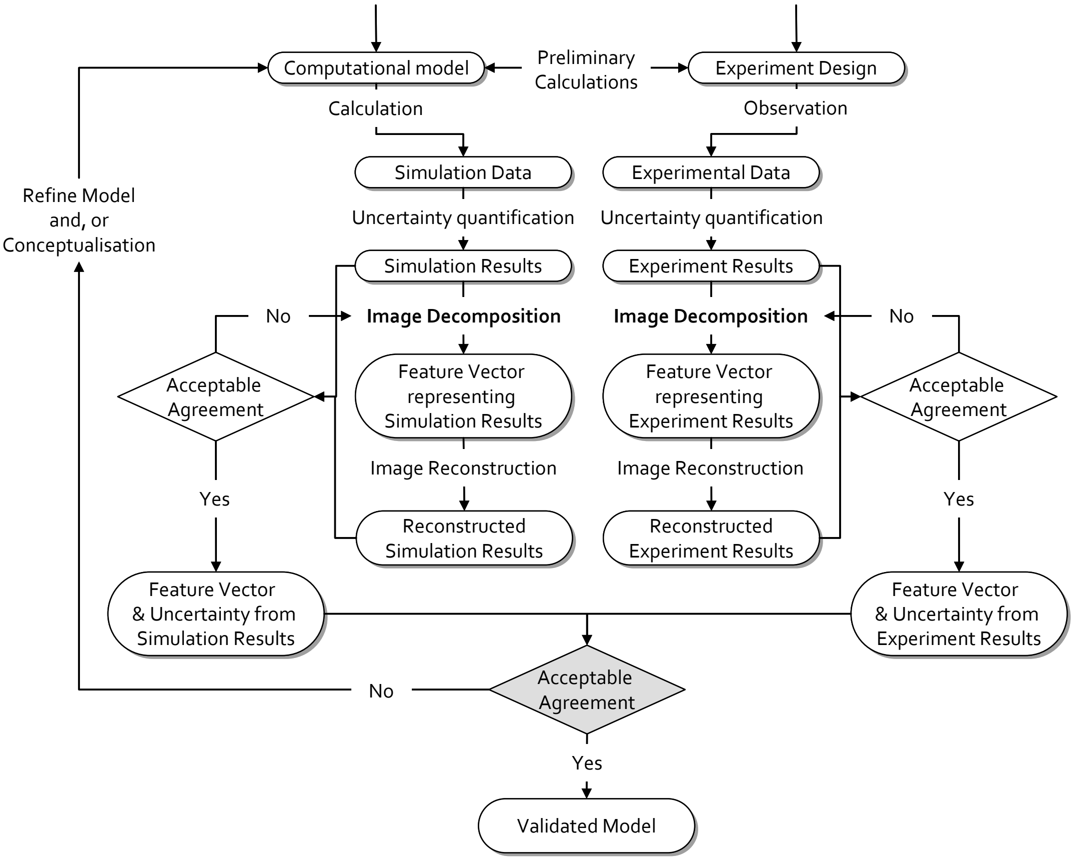

Figure 1 – Flow chart for validation process using image decomposition

The next section describes the process for making such a quantitative comparison. The remainder of this section describes a recommended process for strain field decomposition, assuming the engineering artefact is planar, or near to planar, in the region of interest (ROI); so that the effects of three-dimensional shape and perspective on the view are neglected.

The selection of an appropriate decomposition process for a strain field can generate a set of shape descriptors that are invariant to scale, rotation and translation. This invariance allows comparison of strain fields to be made using their representative shape descriptors regardless of whether the strain fields are in the same coordinate system, have the same scale, orientation, or sampling grid. The only consideration is that the strain fields should share a common region of interest relative to the artefact.

Orthogonal shape descriptors possess the required invariance to scale, rotation and translation. Zernike, Tchebichef and Krawtchouk polynomials give rise to orthogonal shape descriptors. Zernike polynomials have been used to generate shape descriptors for modal shapes in vibration analysis[6] and can also be used to represent strain distributions[7]. However, they are based on a polar co-ordinate system and so are especially appropriate for strain fields with rotational symmetry. Tchebichef and Krawtchouk polynomials are defined on a Cartesian coordinate system and are discrete, so that they are an order of magnitude faster to implement for strain fields acquired from most optical systems of strain measurement[8]. Zernike and Tchebichef polynomials yield global shape descriptors, and it has been found that they do not provide an accurate description of strain fields when there are cut-outs or holes due to the geometry of the artefact present in the image. This issue can be handled by tailoring the Zernike moments to the individual geometry[9]; or using Krawtchouk polynomials; or by performing a fast Fourier transform on the image and then representing the magnitude component of the FFT using either Zernike[10] or Tchebichef polynomials.

The strain image, *I(i, j)* can be decomposed as a series expansion of Tchebichef polynomials, *T(i, j)*

$$I(i,j)=\sum_{k=0}^{N} s_k T_k(i,j) \qquad (6)$$

in which the coefficients s_k constitute the feature vector and are given by

$$s_k=\sum_{i,j}^{n} I(i,j)T_k(i,j) \qquad (7)$$

Note that since the polynomials are dimensionless, all s_k have the same unit as the image I, i.e. strain. The identity Eq.(6) is exactly valid for $N=\infty$ or $N=n$ where n is the number of data points. However, it has been found that no advantage is gained by using a series expansion of order greater than twenty and that eight is usually sufficient. The number of shape descriptors, N, can be limited by including in the feature vector only those with the highest magnitudes, e.g. those

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[6] Wang, W., Mottershead, J.E., Mares, C., (2009) Mode-shape recognition and finite element model updating using the Zernike moment descriptor, *Mech. Systems & Signal Proc.,* 23:2088-21121.

[7] Wang, W., Mottershead, J.E., Patki, A.S., Patterson, E.A., (2010), Construction of shape features for the representation of full-field displacement/strain data, *Applied Mech. & Materials*, 24-25(2010):365-370.

[8] Sebastian, C., Patterson, E.A., Ostberg, D., (2011), Comparison of numerical and experimental strain measurements of a composite panel using image decomposition, *Applied Mechanics and Materials*, 70:63-68.

[9] Wang, W., Mottershead, J.E., Sebastian, C.M., Patterson, E.A., (2011), Shape features and finite element model updating from full-field strain data, *Int. J. Solids Struct*. 48(11-12), 2011, 1644-1657.

[10] Patki, A.S., Patterson, E.A., (2011), Decomposing strain maps using Fourier-Zernike shape descriptors, submitted to *Experimental Mechanics*.

with a magnitude greater than 10% of the magnitude of the maximum shape descriptor[11].

It is important to consider and test the accuracy with which the feature vector describes the original strain field by reconstructing the strain field from the feature vector.

The goodness of fit of the reconstruction of a strain field to the original strain field should be assessed using the average squared residual

$$u^2 = \frac{1}{n}\sum_{i,j}^{n}\left(\hat{I}(i,j) - I(i,j)\right)^2 \qquad (8)$$

where $\hat{I}(i,j)$ *is the reconstructed value of* $I(i,j)$*; and the average residual,* u *should be no greater than the minimum measurement uncertainty, obtained from the calibration of the measurement instrument. In addition, no location should show a clustering of residuals greater than 3u, where a cluster is defined as a group of adjacent pixels comprising 0.3% or more of the total of number of pixels in the region of interest [R4].*

If the reconstruction is found to be unacceptable, then steps should be taken to refine it until it becomes acceptable, and these may include employing a Fourier transform as described above, increasing the order of polynomial representation, selection of an alternative orthogonal shape descriptor, or tailoring of a shape descriptor.

The process of representing the strain field by a set of shape descriptors is performed independently for the results from the model to be validated and from the experiment performed for the purpose of validation, but the identical type and order of shape descriptors must be used, resulting in two feature vectors $(s_E)_k$ and $(s_M)_k$, respectively. The goodness of the representation is described by the residual u, defined in equation (8), which at the same time constitutes the uncertainty $u(s_k)$ of the shape descriptors, s_k. Since the image decomposition is made using orthonormal polynomials, this uncertainty is equal for all $k=1 \ldots N$.

3.3. Correlation of Strain Fields

The shape descriptors representing the strain fields, for identical regions of interest, obtained from the model being validated and the experiment performed for the purpose of validation need to be compared quantitatively. It is recommended that the coefficients (elements of the feature vector) representing the results from the model should be plotted as a function of those obtained from the experiment. If the correlation were perfect then all of the resultant data points would lie exactly on a straight line of a gradient of unity. In practice, this will not occur either due to noise in the data or because the model is a poor representation of the reality of the experiment. The *model can be considered to be a good representation of the reality of the experiment, if all of the data-points lie* *within a band of width* $\pm 2u(s_E)$ *around the ideal line,* $s_M = s_E$ *(see Fig. 2), where* s_M *and* s_E *are the shape descriptors representing the strain fields from the model and experiment respectively; and* $u(s_E)$ *is the uncertainty in the feature vector describing the data from the experiment and should be cited when describing the validity of the model [R5].*

The experimental uncertainty is estimated from the residuals u_E, using equation (8), but must be combined with the calibration uncertainty, i.e.

$$u(s_E) = \sqrt{u_{cal}^2(\varepsilon) + u_E^2} \tag{9}$$

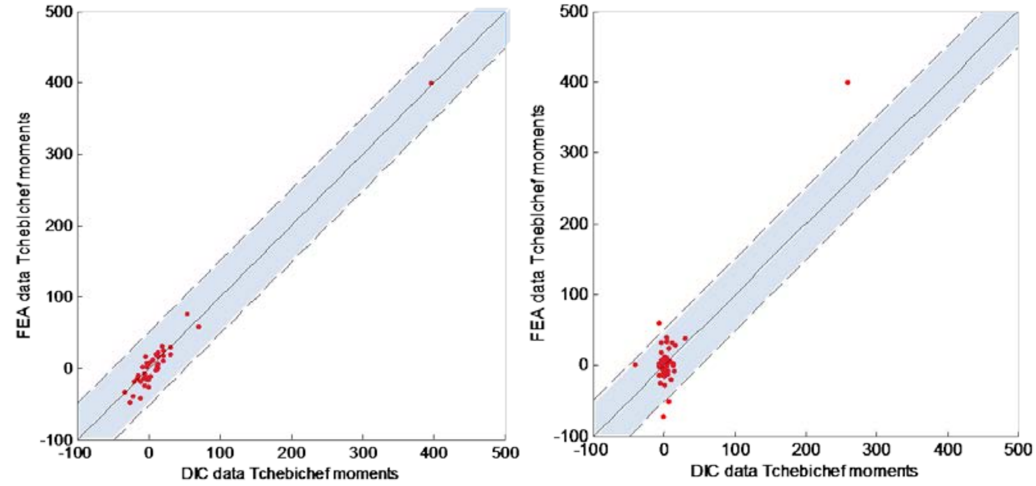

Figure 2 – Shape descriptors representing a strain field from a model plotted as a function of the shape descriptors representing the corresponding strain field from the validation experiment for an acceptable (left) and unacceptable (right) validation, based on whether or not the plotted points (blue circles with a regression line in red) fall within a region (green shading) defined by $s_M = s_E \pm 2u(s_E)$

This process of comparison should be repeated for each loading case for which the fundamental mechanics of the model are changed, e.g. when moving from a linear to non-linear regime, or when the boundary conditions are changed. Consideration should be given to defining a envelope for which the validation holds.

3.4 Conclusion

For the validation process it is essential that a quantitative indication of the quality of experimental data is provided. The measurement uncertainty is the natural basis for this and is used for defining acceptance bands.

It is not the intention that this guideline should provide a definitive or prescriptive methodology for the validation of a computational solid mechanics model. Instead, an objective criterion and a set of associated tools are provided that can be incorporated into a plan or strategy for verification and validation, which is appropriate to the model and its intended uses. The ASME Guide for Verification and Validation in Computational Solid Mechanics[1] provides further guidance on such plans and strategies, so that the procedures described here can be seen as complementary to the ASME guide.

Glossary

Feature vector – a vector containing the coefficients of the shape descriptor employed to describe the image or strain field.

Image decomposition – the process of representing an image, or strain field, at reduced dimensionality, using a shape descriptor.

Shape descriptor – a method to represent the image or strain field such as Fourier descriptors or orthogonal polynomials.

Nomenclature

$d(i,j)$	field of deviations between predicted and measured values
(i, j)	co-ordinates of general point in image
$I(i, j)$	strain value in image
$\hat{I}(i, j)$	Reconstruction of $I(i, j)$
k	index of coefficients, s
N	number of coefficients in feature vectors
s_k	coefficients of polynomials used for shape description
s_E, s_M	feature vector describing data from Experiment & Model
$T(i, j)$	Tchebichef polynomials
u, u_E, u_M	average residual, defined by equation (8), for the Experiment and Model
$u_{cal}(\varepsilon)$	Calibration uncertainty, defined in equation (2)
$u(d)$	Uncertainty associated with measurements in the Reference Material
$u_{model}(\varepsilon)$	Uncertainty in the model
$u_{RM}(\varepsilon)$	Reference material uncertainty
$u(s_k)$	Uncertainty of shape descriptors
$u(s_E)$, $u(s_M)$	Uncertainty in feature vectors from Experiment and Model
$\varepsilon(x_i,y_i)$	strain at i^{th} point (x_i,y_i)

APPENDICES

The two manuscripts attached illustrate the methodology described in this document using components that are relevant to the surface transportation sector. It should be noted that the manuscripts were prepared and published at an earlier stage of the development of the validation procedure, when the definition of the acceptable area in figure 2 had been finalised. Consequently, the definition in these manuscripts is

$$(s_M) = (s_E) \pm 2\sqrt{u^2(s_M) + u^2(s_E)}$$

This definition was modified because the uncertainty in the model, SM is often unknown and because its inclusion allowed a model with a large uncertainty to be easily validated which seemed counter intuitive or contrary to good engineering practice.

The paper were published in the proceedings of the International Conference on Advances in Experimental Mechanics: Integrating Simulations and Experimentation for Validation held in Edinburgh in September 2011 organised by the BSSM and SEM. They should be cited as:

Lampeas, G., Pasialis V., Siebert, T., Feligiotti, M., Pipino, A., 2011, Validation of impact simulations of a car bonnet by full-field optical measurements, *Applied Mechanics and Materials*, 70:57-62.

Sebastian, C.M., Patterson, E.A., Ostberg, D., 2011, Comparison of numerical and experimental strain measurements of a composite panel using image decomposition, *Applied Mechanics and Materials*, 70:63-68.